Controlling Wave Phases in Curved Space for Light

Y. Liu[1], **B. Vial**[2], **Z. Mao**[1], **K. Peng**[1], **and B. Zhou**[1]

[1]School of Physics and Electronic Science, Hubei University, Wuhan, Hubei 430062, China
[2]School of Electronic Engineering and Computer Science, Queen Mary University of London
London E1 4NS, England, UK

Abstract— In this submission of abstract, we review a general methodology to manipulate the amplitude of an electromagnetic wave in a pre-defined way, without introducing any scattering. This leads to a whole class of isotropic spatially varying permittivity and permeability profiles that are invisible to incident waves. The theory is illustrated through various numerical examples, including the non-magnetic case. Also some new work on manipulating the wave phases is extended in the hope to develop this methodology further.

Introduction: Transformation optics [1, 2] is a powerful analytic tool to design impedance-matched material, which gives rise to scattering-free wave solutions up to designers will. However, this method inevitably requires the material parameter to be anisotropic, which complicates its manufacturing process. Therefore, an isotropic material to achieve the required wave solution may solve this issue. Here in this conference contribution, we try to reveal one method to serve the purpose to cater for designers requirements for electromagnetic waves.

We will demonstrate a method to design isotropic material parameters in an optical inverse problem. We will start from vectorial Helmholtz equations [3, 4] and show how one can find some exact wave solution in a corresponding inhomogeneous medium, where light wave and medium come in pairs. After that we exploit further the idea to seek the pair of wave solution and material parameters: spatial profiles of permittivity and permeability.

We demonstrate examples of how to control wave front [5–9] — whether maintaining it or varying it at our will. For sake of simplicity, our design herein are all two-dimensional cases. However, this theory is applicable also in three-dimensional. Another instance we will show is a phase converter from a cylindrical wave to an planar wave as output. Our method is a general analytic tool to avoid unwanted scattering due to gradient-index profile of materials [7, 10]. This methodology can be extended to a general design tool for arbitrary wave solution, which gives spatial-varying, isotropic profiles of material parameters.

Equations: We consider here lossless and possibly dispersive isotropic material characterized by their relative permittivity ε, and relative permittivity μ, and assume that theses quantities are invariant along z. Let us consider that the electric (resp. magnetic) field is linearly polarized along the z axis $\mathbf{E} = E_z\mathbf{z}$ (resp. $\mathbf{H} = H_z\mathbf{z}$), so that we stick with the scalar wave equation.

Under these conditions, Maxwell equations can be recast as the scalar wave equation:

$$\nabla \cdot \left(\frac{1}{\xi} \nabla F \right) + k_0^2 \chi F = 0 \tag{1}$$

where $F = E_z$, $\chi = \epsilon$, $\xi = \mu$ for the TE case and $F = H_z$, $\chi = \mu$, $\xi = \epsilon$ for the TM case. By writing each field in polar form as $F = Ae^{\mathrm{i}\phi}$, we intentionally separate its amplitude A and phase ϕ. Substituting F in the wave Equation (1) and equating the real and imaginary parts reads:

$$\nabla \cdot \left(\frac{A^2}{\xi} \nabla \phi \right) = 0 \tag{2}$$

$$(\nabla\phi)^2 - k_0^2 \chi \xi - \frac{\nabla^2 A}{A} + \frac{\nabla \xi}{\xi} \cdot \frac{\nabla A}{A} = 0 \tag{3}$$

The two equations above are at the heart of the current method.

Conclusion: In this contribution, we will demonstrate a method how one designs spatial-varying, isotropic materials for EM waves, which turns out scattering-free.

ACKNOWLEDGMENT

The authors were sponsored by Natural Science Foundation of China (NSFC11804087), EPSRC grant (EP/I034548/1), Department of Science and Technology (2018CFB148, 2020CFB266), and Educational Commission of Hubei Province (Q20201006). L. Y. thanks Prof. Xu Donghui for useful discussion.

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